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THE SCALING OF UNDERWATER EXPLOSION PHENOMENA

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by

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ABSTRACT: The objective of this oral presentation was to summarize the problems and the semantics of scaling in a concise form without use of mathematical developments. The well known cube-root scaling is shown to be a consequence of Mach's similitude, the fourth-root scaling, of Froude's similitude. Requirements which must be satisfied for each of these scaling rules are discussed. Scaling of gravitational effects on underwater explosions (bubble behavior) is not possible by means of model tests conducted in a river or a pond, but requires test tanks in which the air pressure above the water can be reduced or the acceleration of gravity increased. The scaling of surface tension, vapor pressure, and viscosity are discussed. Model tests on damage to targets are not considered. A short discussion on the concept of approximate scaling is included.

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The following paper is a transcript of an oral presentation given at the meeting of the APEX Committee on 11 May 1961. The APEX Committee is an advisory group sponsored by the Naval Ordnance Laboratory, White Oak, the Naval Radiological Defense Laboratory, and the David Taylor Model Basin. Since the scope and limitations of scaling are a subject of general interest, a short summary which clarifies the terminology and outlines the methods might be welcome and is made available in this report. The work was done under Task No. RE01-ZA732/212 9/F008-21-003.

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INTRODUCTION

Model tests and scaling play an indispensable role in explosions research. In the following paper, the attempt is made to summarize the fundamentals of scaling and to clarify the terminology. It will be further attempted to highlight the possibilities as well as the limitations which are inherent to the method of scaling.

SEMANTICS

It is a well-established fact that many scientific terms as used in physics, chemistry, and other fields have a different connotation in the professional and in the every day language. This includes the word "scaling".

In every day language, to scale means (among others) to weigh, compare, and also to reduce in size according to a fixed ratio: for instance, the prices were scaled down 5%. The term "scaled experiment" is occasionally used as a synonym for small scale experiment. The latter usage is clearly objectionable, because in reference to scientific experiments the term scaling means more than to reduce the scale. If we speak of scaling laws, or if we say, "the attempt is made to scale the base surge by means of a model test", we definitely mean more than just a reduction of size. We always imply that, despite the reduction of size, results applicable to the full scale event will be obtained. Hence, it is implied that the model test will reproduce in some way the full-scale phenomenon we want to study. In other words, we imply that there is a similarity between the full-scale and the model test. The word "model" actually implies similarity as it is evident if we recall terms such as "model of a ship", "model railroad".

In our case we cannot be satisfied with a weak, qualitative similarity. Model tests are made to provide quantitative answers, numbers. Such quantitative answers can be obtained by model tests, if a specific, rigorous type of similarity is established which we call "similitude".

The objectives of the "scaling analysis" are to determine (a) whether or not similitude can be achieved in a model test, and if so (b) what the scale factors are.

If it turns out that the answer to (a) is negative, it is said that this phenomenon "cannot be scaled". Thus, strictly speaking, the term "scaled test" should imply that point (a) has been investigated and a positive result found. As mentioned, this is often at variance with common usage.

To summarize: the word scaling implies a specific and rigorous type of similitude. The word "scaling" is and has been used in this meaning not only in explosion research but also in other fields, in particular in hydrodynamics. Many books on our subject use the term "modeling". Scaling and modeling are synonyms. However, they are not commonly used interchangeably. It seems that the word "scaling" has replaced the word "modeling" in certain fields.

REQUIREMENTS FOR SIMILITUDE

After we have established that similitude is a must for any meaningful model test, we will now proceed to the methods or the criteria which will assure similitude. These methods are called scaling laws, the laws of similitude, or modeling rules, or the theory of models. They all mean the same thing (See Appendix of this report.).

Explosions, as any other physical phenomena, are complex processes. This means there are many different effects which influence the sequence of events. Ideally, there must be similitude of all of these effects. Hence, there is not one, but a great number of similarity requirements which must be satisfied.

THE THREE BASIC SIMILITUDE REQUIREMENTS

In any non-static model test, at least three basic requirements for similitude must be satisfied. These requirements are necessary but not always sufficient to assure similitude for the phenomenon to be studied. These three basic requirements refer to geometric, kinematic, and dynamic similitude.

BASIC SIMILITUDE REQUIREMENTS	
Geometric Similitude	
Length Scale Factor	λ
Kinematic Similitude	
Velocity Scale Factor	$\varphi = \lambda / \tau$
Acceleration Scale Factor	$\alpha = \lambda / \tau^2$
Dynamic Similitude	
Pressure Scale Factor	$\pi = \rho \varphi^2$
Energy Scale Factor	$\varepsilon = \pi \lambda^3$

Figure 1

Geometric Similitude. There are several ways to express the requirement of similitude in a quantitative way. Here, we will use the concept of the scale factor for this purpose. In Figure 1 the note "length scale factor, λ" is made under the first heading. This is to indicate the following concept. If we reduce all dimensions of a given configuration by the same factor, we obtain a smaller configuration which is geometrically similar. We call the factor used, the length scale factor λ. Everybody is familiar with the meaning of this magnitude: If we talk about a model test in a scale 1:10, the length scale factor is 0.1. Of course λ must have the same value in all three directions of the coordinate system and λ must be constant with time.

> To repeat: if we meticulously apply the rule, that every item of the full scale prototype must be present in the model and that all dimensions of these items must be reduced by λ, then we are assured of geometric similitude.

It is clear that geometric similitude is a prime requirement for model tests. Still, often geometric similitude is either deliberately omitted or it turns out to be more difficult to satisfy than it may appear at the first glance.

Example 1. If we want to study underwater explosion damage against submarines we may consider to build a model of the target and to conduct the test on a small scale. When building such a model, besides geometric similitude other rules for similitude must be observed which are not of interest here. But, if one is convinced that, for instance, the conning tower does not influence the damage pattern, one may as well omit this item in the model and one may be satisfied with approximate geometric similitude. Obviously, there is no need to reproduce details which apparently would not influence the result obtained.

Example 2. In a free water explosion, geometric similitude of the explosive charge is an obvious necessity for similitude. However, it turns out that it is difficult to make exactly similar charge configurations. The firing cap as well as the booster have a certain size which cannot be indefinitely reduced. Even if this were possible, one would not do so, because such small detonators will not assure a high order detonation. Therefore, in most practical cases, the detonator will be the same in the full-scale charge and in the model. Obviously, this is a violation of the requirement of strict geometric similitude. Fortunately, in most cases, this violation is not serious. Another characteristic length of an explosive charge can spoil the requirements of similitude. This is the length of the reaction zone. If the same explosive is used in the full-scale and in the model test, this length is not reduced by the scale factor λ, as it should be. The only alternative for strict similitude would be to use a different explosive in the model test which again, in principle, is a violation of the similitude requirements as will be seen below.

Example 3. There are cases where a deliberate deviation from similitude is introduced for the purpose to account for effects which cannot be scaled. In this case one speaks of "distorted models". For instance in models of rivers which are supposed to predict the propagation of floods and similar phenomena, the height of the water above ground is usually not reduced in the same scale as the horizontal dimensions, or else the stream would be so shallow that adhesion and surface-tension become the leading factor. Such methods, do not comprise what one may call true scaling, because additional information, such as theoretical calculations or empirical data or formulae, is needed for the evaluation of such tests. Finally, one sometimes speaks of a dissimilar model which is a synonym of "analogue", for instance an electrical circuit which simulates oscillations of a mechanical system.

Kinematic Similitude. The next requirement is that of kinematic similitude, i.e., the similitude of motions. Here, the time scale factor τ is introduced. The similitude requirements yield expressions for the velocity scale factor and the acceleration scale factor. The expression for the velocity scale factor must not be interpreted that velocity is equal to distance divided by time or that the velocity is constant with time. The expressions shown in Figure 1 are a result of the constancy of the scale factors and amount to nothing more than the following simple transformation: Consider a velocity occurring in the model test u_m, then

$$u_m = \frac{dx_m}{dt_m} = \frac{d\lambda x}{d\tau t} = \frac{\lambda}{\tau}\frac{dx}{dt} = \varphi u.$$

We have used the subscript m to designate magnitudes which refer to the model, whereas magnitudes referring to the full scale are those without subscript.

Kinematic similitude is an extension to velocity and acceleration of the principles explained for the geometric similitude. For kinematic similitude, any and all velocities occurring in the model test must be reduced by the velocity scale factor φ. Of course this applies to velocities or accelerations which occur at corresponding locations and at corresponding instants of time:

$$u_m\,(t_m, X_m, Y_m, Z_m) = \varphi\, u\,(t, X, Y, Z),$$

or in words: The velocity in the model test u_m which occurs at the time t_m and at a point having the coordinates X_m, Y_m, Z_m, is φ times the velocity in the full scale at the time t and the location X, Y, Z. The coordinates are interrelated by

$$t_m = \tau t$$

$$X_m = \lambda X$$

$$Y_m = \lambda Y$$

$$Z_m = \lambda Z\,.$$

Dynamic Similitude: Requirements for dynamic similitude can be derived from Newton's law. This requirement assures that interactions between driving forces and inertial forces are similar. For our purpose the introduction of the pressure scale factor π is convenient, Figure 1. $\tilde{\rho}$ denotes the density scale factor, i.e., the ratio of the densities occurring in the model and the full scale tests. (There are other scale factors connected with dynamic similitude which we do not need for our purpose.)

For completeness, the energy scale factor is listed, although it does not necessarily belong under this heading.

Magnitudes Available in the Scaling Analysis. We have so far listed scale factors for velocity, acceleration, pressure, and energy, but it appears that we can freely dispose of only two scale factors, namely, the time scale factor τ and the density scale factor $\tilde{\rho}$.

The length scale factor λ must be excluded because this magnitude is to be considered as an independent variable. It is dictated by practical considerations, namely the scale of the model test. For instance, if we decide to simulate a 10 KT nuclear explosion by means of a 1,000-lb charge, the length scale factor is fixed. It is now the purpose of the scaling analysis to find relationships for the other magnitudes such as velocity, pressure, etc.

If we consider underwater explosions, the choice is further reduced to the time scale factor τ alone, if the model tests are made in the same medium as the full scale tests, namely in water. This is a very common arrangement which we may call field model test. We will make the assumption of equal medium for the time being and will discuss later the possible advantages of using other liquids for the model explosion. If water is used in the model test, $\tilde{\rho} = 1$ and only τ remains free to choose.

FURTHER REQUIREMENTS FOR SIMILITUDE

The list of similarity requirements, Figure 1, is not complete by far. This is evident since so far only one material constant is listed, namely the density ρ. But, there are many other material constants which may affect the explosion process such as compressibility, viscosity, surface tension, vapor pressure, and finally gravity.

The procedure to be followed is well-known. The similarity requirements can be obtained from the pertinent differential equations by means of a similarity transformation. The method is treated at length in the literature and it is not necessary to reiterate it here. We can make direct use the results in the same way as the rules for differentiation or integration are applied without recourse to the original derivation. The scaling analysis has resulted in the rule that for similarity of a specific effect a characteristic number (i.e., a dimensionless magnitude) must have the same value for the model test and the full scale condition. It has become customary to give these characteristic numbers the names of famous scientists. Figure 2 shows a list of a few of the numbers which may have a bearing for explosions in free water (i.e. in absence of targets.). If similitude of the effects listed is desired, the pertinent characteristic numbers must have the same value for the model and the full scale. Figure 2 lists such numbers. There, u refers to a velocity, L, to a length, T, to a time, and P, to a pressure. Here the question may be raised which velocity, length, time, or pressure should be inserted into these numbers. Strictly speaking, it does not matter, e.g. any velocity may be used so long as corresponding velocities (in the sense discussed above) are considered in the full scale and in the model test. Commonly one would use a typical magnitude, for instance the peak velocity.

Similitude of Compressibility Effects

Mach Number $= \frac{u}{c}$ $\qquad \varphi = \tilde{c} = c_m/c$

Similitude of Gravitational Effects

Froude Number $= \frac{L}{gT^2}$ or $\frac{u^2}{gL}$ $\qquad \alpha = \tilde{g} = g_m/g$

Similitude of Evaporation and Condensation Effects

Thomas Number $= P/P_{vapor}$ $\qquad \pi = \tilde{P}_{vapor} = (P_{vapor})_m/P_{vapor}$

Similitude of the Effect of Surface Tension

Weber Number $= \frac{PL}{\vartheta}$ $\qquad \pi\lambda = \tilde{\vartheta} = \vartheta_m/\vartheta$

Similitude of the Effects of Viscosity

Reynolds Number $= \frac{uL}{\nu}$ $\qquad \lambda^2/\tau = \tilde{\nu} = \nu_m/\nu$

Figure 2
Additional Requirements for Similitude

In Figure 2, the characteristic numbers are also given in terms of the scale factor. It is seen that in these relationships the scale factor for sound velocity $\tilde{c}$, the scale factor for gravity $\tilde{g}$, the scale factor for the vapor pressure $\tilde{P}_{vapor}$, the scale factor for surface tension $\tilde{\vartheta}$, and the scale factor for the kinematic viscosity $\tilde{\nu}$, occur. For field tests all these scale factors are unity, because of the same medium and the same gravitational acceleration for the model test and the full scale.

THE SCALING OF EXPLOSION PHENOMENA

Phenomena which are associated with explosions include first of all shock waves or blast waves. These pressure waves are a direct consequence of the compressibility of the medium. Therefore, in most model tests on explosion phenomena, it is important to make sure that there is similitude of the effects of compressibility. Hence, the Mach similitude requirement must be satisfied.

In explosions research, we commonly do not refer to the Mach number but to the so-called cube root scaling which is the result of Mach's similitude.

It is simple to derive the cube root scaling from the relations listed in Figures 1 and 2. If we consider field tests, i.e., the same medium in the full scale and in the model, then $c_m = c$, and hence, from Mach's similitude requirement, $\varphi = 1$. This result could have been obtained without making recourse to Mach's number, if the rules of similitude are consequently obeyed. We have previously stated that any and all velocity should be reduced by the velocity scale factor, φ. This also applies to the velocity of sound. If we decide to make the model tests in the same medium as a full scale test, we have fixed the velocity scale, namely $\varphi = 1$.

Going back to Figure 1 we obtain immediately the following relationships:

Velocity Scale	$\varphi = 1$	
Time Scale	$\tau = \lambda$	for field mode tests
Pressure Scale	$\pi = 1$	$\tilde{\tau} = 1 \quad \tilde{\rho} = 1$
Energy Scale	$\varepsilon = \lambda^3$	

Since the energy of the explosive charge is proportional to its weight W, the expression for the energy scale factor ε yields

$$\left(\frac{W_m}{W}\right)^{1/3} = \lambda = \left(\frac{X_m}{X}\right) = \left(\frac{Y_m}{Y}\right) = \left(\frac{Z_m}{Z}\right)$$

and we obtain for the

Pressure $\quad p_m\,(t_m, X_m, Y_m, Z_m) = p\,(t, X, Y, Z)$

Velocity $\quad u_m\,(t_m, X_m, Y_m, Z_m) = u\,(t, X, Y, Z)$

Time $\quad t_m = \left(\frac{W_m}{W}\right)^{1/3} \cdot t$

This scaling rule has various names, the most common designation being "cube root scaling". Sometimes it is also called Hopkinson's law or Hilliar's law. Another rather appropriate expression is "isovelocity scaling". As we have seen, these laws are nothing more than the consequence of Mach's scaling when applied to explosion phenomena.

RESULTING REQUIREMENTS

It is important not to overlook certain implications which result from the requirement of consistent similitude. It is self understood that there must be geometric similitude. If the effect of the bottom of the sea is to be studied, the bottom material must have the same density and the same sound velocity as in the full scale. Since the sound velocity changes with pressure, $\pi = 1$ and $\varphi = 1$ requires that the sound-velocity-pressure relationship must be identical for the model and the full scale.

Similarly, the explosive used in the model test must be the same as for the explosive of the full scale prototype, that is the following properties must be the same.

(a) The density of the explosive as well as that of the gaseous reaction products,

(b) The energy of detonation per unit weight,

(c) The detonation pressure and detonation velocity, and

(d) The sound velocity at each point of the isentropic expansion curve.

Since the sound velocity corresponds to the inclination of the isentropic pressure-density curve, (d) is equivalent to the requirement of an identical isentropic expansion curve. In other words the thermic and caloric equation of state must be the same. Excepting a few special cases, requirements (a) to (d) can be practically satisfied only by one and the same explosive.

AN ILLUSTRATION OF THE SCOPE OF SCALING

At this point an example may illustrate the extent for which scaling can provide answers. Consider the change of the shockwave peak pressure p with distance R. The result obtained above amounts to

$$p = f\left(\frac{W^{1/3}}{R}\right),$$

where f is an unspecified function. For the peak pressure of a TNT explosion in water, the complete expression is the following:

$$p = 21,600\left(\frac{W^{1/3}}{R}\right)^{1.13}.$$

The contribution of the scaling analysis is the establishment of the variable $W^{1/3}/R$. Scaling analysis cannot provide the functional relationship for f. This must be found either by experiments or theory. We are reasonably sure that the exponent for W must be 1/3 and we should be inclined to reject experimental results which indicate different values for this exponent. However, the exponent 1.13 in the above equation is an experimental result which can never be obtained from the scaling analysis alone. The great advantage of the scaling analysis is that it is not necessary to cover the entire range of charge weights W and distances R which might be of interest. The experimental effort in the determination of the function f is greatly reduced by the elimination of these two variables and concentration on the single variable $W^{1/3}/R$.

THE LIMITATIONS OF SCALING

We will now proceed to the most important conclusion to be drawn for the scope of the scaling analysis. The assumption of equal media in the full scale and in the model test left only one variable, τ, to dispose of in the scaling analysis. Using the Mach similarity requirement, τ is determined to be $\tau = \lambda$. Therefore, there is no further variable available to satisfy additional similitude requirements. A glance at Figure 2 shows that most of the other requirements contradict each other. It is impossible to satisfy these at the same time: In particular, Mach's and Froude's scaling requirement cannot be satisfied simultaneously. In fact, the requirement for similitude of compressibility, gravitational effects, surface tension, and viscosity are not compatible with each other. We have here a grave drawback of the scaling technique. We cannot satisfy all requirements for similitude, and, therefore, can never achieve complete ideal similitude. We are restricted to such phenomena which are predominantly governed by only one of these effects. Thus, we can satisfy either Mach's or Reynolds', or Weber's requirement. To make the situation worse for the study of underwater explosion phenomena, it turns out that Froude's similitude cannot be achieved in field tests, even if the other requirements are ignored.

THE EFFECT OF GRAVITY ON UNDERWATER EXPLOSION PHENOMENA

Gravity affects strongly the behavior of the underwater explosion bubble. The size of the bubble produced by a charge weight of, say, 100-lbs is so large that the effect of buoyancy becomes noticeable despite the relatively short time of the bubble pulsation. For larger charge weights and, in particular, for nuclear explosions this effect is, of course, even more pronounced.

Buoyancy is an effect which results from gravity. Buoyancy is a consequence of the increase of the hydrostatic pressure with depth. It is the resultant force, if the pressure is integrated over the surface of a submerged body. There would be no buoyancy if the hydrostatic pressure around the body were constant. Since the change of the hydrostatic pressure with depth is clearly a consequence of gravity, Froude's law is applicable if effects of buoyancy are to be studied.

Gravity has a very profound effect on the behavior of a pulsating bubble. It produces an upward motion of the bubble center, the so called gravity migration. Also, it produces a change of shape of the bubble and a collision of the bubble interfaces when the bubble approaches its minimum size. The strength of the migration and the details of the change of shape and of the impact of the interfaces depend strongly on the interplay between buoyancy and the time for which it is effective, i.e., the period of the pulsation. We will now derive the conditions which must be satisfied for an experimental study of this effect in a small scale.

SCALING OF GRAVITATIONAL EFFECTS IN UNDERWATER EXPLOSIONS

Since it is not possible to satisfy Mach's and Froude's similarity requirement simultaneously, one may attempt to ignore the effects of compressibility and try to reproduce the phenomena caused by gravity.

Obviously, the neglect of compressibility is rather serious for explosion phenomena. However, as an approximation one may assume that the underwater explosion bubble is not affected by compressibility. This does not hold for the moment where the bubble starts to expand or where it contracts to its minimum, but, it is a good approximation for the relatively long time where the pressure inside the bubble is small.

Froude's similitude requirement leads to the following relationship between length scale factor and time scale factor:

$$\lambda = \tau^2.$$

Again, this result could have been obtained by considerations of consistent similitude, namely that the scale factor for accelerations must be the same for all accelerations including the acceleration of gravity.

From Figure 1 we obtain with $\tilde{\rho} = 1$

Velocity scale factor $\varphi = \lambda^{1/2}$

Pressure scale factor $\pi = \lambda$

Energy scale factor $\varepsilon = \pi \lambda^3 = \lambda^4.$

The last expression is the basis of the so-called 4th root scaling of underwater explosion phenomena under the effect of gravity. However, certain precautions must be observed in the use of this scaling law as will be seen by consideration of the pressure scaling.

The result that the pressure scale factor is equal to the length scale factor reflects the important situation that the hydrostatic pressure increases with depth and, therefore, is proportional to a length. However, this rule must be applied to all other pressures. For instance, the detonation pressure must be also reduced proportional to the length scale of the model test. Of particular importance is the reduction of the atmospheric pressure above the water. As a consequence, the scaling requirement for gravity must be expressed by two relationships: One referring to the length scale, the other referring to the pressure scale.

It is customary to express the total hydrostatic pressure in the water in terms of the hydrostatic head Z = 33 ft + depth. Here, 33 ft represents the head of the atmosphere in ft of sea water. Then, the gravity scaling requirement can be formulated as follows: For similitude of gravitational effects the two magnitudes

$$\frac{W^{1/4}}{D} \quad \text{and} \quad \frac{W^{1/4}}{Z} \tag{1}$$

must have the same value for the full scale and model test. Here, D denotes the depth of explosion. It is obvious that this requirement cannot be rigorously satisfied in a field test. It is the basis of the vacuum tank technique where explosions are made in a closed container and where it is possible to reduce the air pressure above the water. To obtain scaling relations for this purpose, one may write the scaling requirements as follows:

$$\frac{W^{1/4}}{D} \quad \text{and} \quad \frac{W^{1/4}}{D+x\,33} \, . \tag{2}$$

If in the vacuum tank the magnitude x is reduced proportionally to the length scale, both requirements listed above can be satisfied. However, it is not possible to satisfy the scaling requirements for similitude of gravitational effects in a field test. A possibility might be to make the tests in a mountain lake at a great height. All lakes known and suitable for this purpose are by far not at an altitude which would be necessary for scaling of explosions of practical interest. For nuclear explosions the pressure must be reduced to such a low value that the vapor pressure of water is approached. In this case the bubble would behave similarly as if it would pulsate in water which is near to its boiling point and scaling fails for this reason. (In a test tank, the use of oil instead of water prevents this difficulty.)

APPROXIMATE SCALING OF GRAVITY IN FIELD TESTS

Requirement (1) can be approximately satisfied in a field model test if the depth of the model explosion is large in comparison to 33 ft. This is a valid approximation, but it amounts to a situation where the effect of gravity becomes negligibly small.

Another approximation is to ignore $W^{1/4}/D$ and to satisfy equality of $W^{1/4}/Z$ only. This means to ignore geometric similitude of the bubble-water surface configuration. Since the effect of the water surface on the bubble behavior is small, if the center of the bubble is, say 1.5 to 2 times the maximum bubble radius below the original water surface this method constitutes a good approach in such cases. From the practical point of view, this method is again limited because for small λ (i.e. for large explosions to be simulated by relatively small charge) the depth of explosion of the model becomes so shallow that the bubble is either too close to or, in extreme cases, even above the level of the water surface. Also, it must not be overlooked that for such tests the explosive for the model has to be modified in such a way that it has the same density and specific energy (calories/gram) as the full scale explosive, but produces a pressure which is smaller by the amount of the length scale factor λ.

THE HIGH GRAVITY TANK TECHNIQUE

It is possible to achieve both Mach's and Froude's similitude if the model test is carried out in an accelerated container. If the scale factor of "gravitational acceleration", $\tilde{g}$, can be made equal to $1/\lambda$, we obtain all relationships previously derived for the cube root scaling. The great advantage of such a test arrangement is the possibility to satisfy not only Froude's similitude but also Mach's and Thomas's similitude. This means that in a high gravity tank evaporation and condensation phenomena are scaled together with those of the gravity. This is of particular significance for the study of two important explosion phenomena, namely, the cavitation produced by the interaction of the shock wave with the water surface and for the study of the behavior of nuclear explosion bubbles. These bubbles are filled with steam and not with permanent gases as in the case of ordinary explosives. Since the condensation of this steam is strongly affected by the bubble behavior, which in turn depends on the gravitational effect, a simulation of these phenomena is only possible in an accelerated test tank.

MODEL TESTS OTHER THAN FIELD TESTS

The considerations made so far indicate the limitations of the scaling technique as far as simultaneous similitude of different effects is concerned. In the case of gravity, it was even impossible to simulate its effects in a field test. But, dropping the field test technique made scaling possible, either in a vacuum tank or in the high gravity tank. Therefore, the question may be raised: Would it be possible to achieve similar advantages by making the model tests not in water but in another fluid? Of considerable practical interest for underwater explosions is the scaling of the surface tension (Weber's similarity), because it has a notable effect on the formation of the spray dome and other surface phenomena. For the same reason Reynold's scaling would be desirable. A study of the properties of various liquids which could be used for this purpose revealed that none of those show the variation in surface tension and viscosity which would be necessary to satisfy the requirements or even improve the deficiency noticeably.

WHY SIMILITUDE?

Since it appears to be so difficult to satisfy the requirements for similitude, the following question is pertinent at this point: Why are we so anxious about similitude? Are there other ways to make use of small scale tests which do not satisfy the requirements of similitude?

Let us first realize the advantages of a test for which the similitude requirements are satisfied:

Simplicity. All that is needed to obtain full-scale information from the model tests is to divide the magnitudes measured by the pertinent scale factors. Thus a simple change of the scale in the model results produces the full scale data.

Confidence. If there is appropriate similitude, a model test is truly equivalent to the full scale experiment. Model testing of this kind belongs to the exact methods of the physical sciences.

Therefore, it is highly worthwhile, in fact necessary, for everyone who plans an experiment to reflect on the scaling requirements and to strive that they are satisfied as far as possible. This need becomes even more apparent, if we realize that similitude is the only possibility by which quantitative full scale information can be directly obtained from a small scale test. Here, the emphasis lies on "direct" which means: without use of additionally obtained information, such as that from other full scale tests or mathematical theory.

APPROXIMATE SCALING IN GENERAL

Since the fact remains that many underwater explosion phenomena of important nature, in particular those which are connected with the behavior of the pulsating bubble cannot be appropriately scaled, it remains to investigate other possibilities of using small scale explosions to obtain the desired full scale results.

Approximations and idealizations of the actual phenomena by means of simplified concepts are a tool used everywhere in physics. Nobody would seriously attempt to solve the Navier-Stokes equations for a compressible viscid fluid in the case of a slow, regular motion of water. In this case the approximation of an ideal incompressible liquid will yield sufficiently accurate results. Moreover such approximations are also made in cases where the conditions are by far not as clear cut and where the effect of compressibility or viscosity may have an influence. Depending on the situation often excellent results are obtained by this way.

However, it requires experience and skill and a good feeling for the nature of the problem to judge, if and how far such approximations are permissible. Therefore, it is necessary to have a thorough knowledge of the nature of the problem if such approximations are made.

There are three methods which can be used in the case where similitude cannot be achieved or where it is questionable. These are (a) the extrapolation method, (b) the simultaneous attack of a problem by means of theory and experiment, (c) the separation method.

<u>The extrapolation method</u> is promising if the effect which is not scaled in the model test does not have too great an influence on the process. In this case several model tests are made at a different scale. Since scaling is not exactly observed, the reduced results obtained will be different and will be a function of the length scale. A plot of the results versus the scale in which the model test is performed gives some indication of the importance of the neglected effect and often permits an extrapolation to the full scale condition. But, one can be sure of such an extrapolation only if the variations are small.

<u>Theory</u> can be used to calculate and predict the phenomena for the model test as well as for the full scale condition. A comparison between the results of the model test and theory will establish the confidence in the theoretical treatment or will permit to improve this theory until satisfactory agreement is obtained. After such a thorough check, application of the theory to the full scale can be made without hesitation.

The separation method is used in cases where two effects, for instance, that of gravity and viscosity, do not strongly affect each other. A classic example is a study of the drag of ships in a towing tank. The resistance of a ship consists of two portions, that which is caused by the wave formation on water surface and the other which is caused by viscous friction. The first is governed by Froude's scaling, the second by Reynolds' scaling. In the towing tank Froude's scaling can be satisfied but not Reynolds' scaling. Therefore, the drag measured in the towing tank must be corrected. The classic way was to account for the skin friction by means of a theoretical formula. With the use of this formula the skin friction of the model is eliminated so that only the wave resistance remains. This can be scaled to the full scale condition and finally the full scale skin friction is added as obtained from the formula.

In underwater explosion research, shock wave phenomena and bubble phenomena are such effects which one may approximately consider to be independent of each other. Therefore, if a complete picture of an explosion is desired, shock wave measurements could be made using the cube root scaling law and the bubble phenomena could be observed in a vacuum or gravity tank using Froude's scaling. After each of these effects has been scaled up to the full scale condition, a superposition will result in the complete picture of the phenomena.

CONCLUSION

Scaling is an important, in fact, indispensable tool in the study of underwater explosion phenomena.

The great advantages of this method are simplicity and confidence in such cases where it can be properly applied. The great drawback of the method is that it is not always possible to satisfy the requirements of scaling.

APPENDIX

There were no comments by the members of the APEX Committe to this presentation. However, after the meeting, discussions with Mr. Aronson (NOL) and Dr. Focke (APEX member) brought out a point worthwhile to be added to this report.

It was stated that although the terms "scaling" and "modeling" are sometimes used interchangeably, and although the word "scaling" has replaced the word "modeling" in certain fields, it is believed desirable to give them separate meanings for scientific useage. The difference was formulated as follows:

"Scaling is seen to exist in physical experiments when the phenomena of interest can be interrelated by the same physical laws regardless of the magnitude of the experiment. By such a definition all laws of nature are, of course, scalable and scaling never fails: although man may fail in his ability to scale.

"Modeling, on the other hand exists when it is necessary (because of man's failure to scale) to introduce compromises in experiments, to obtain results.

"Both terms imply similitude. Scaling means exact similitude in terms of dimensional analysis of all factors, and modeling means a degree of similitude less than exact, a compromise which is adequate for the problem involved. One may, therefore, consider that scaling never fails but perhaps never exists except as a concept. Modeling may fail as the compromises required are too extensive to achieve the degree of similitude required for the problem."

It seems that notions of this nature are widespread and their justification is not only apparent but clearly convincing. In particular it may be pointed out that in common language the term "model" never implies strict similitude as it does if we use this term in the problem of scaling. (Compare again the terms model railroad, model submarine. Furthermore, the terms "similar model", "distorted model" and "dissimilar model" point in the same direction.)

However, there are two objections against the above formulations.

(1) It can be readily demonstrated that the term "model" or "modeling" usually refers to exact similitude throughout the scientific literature if the opposite is not specifically stated. This holds not only for the English but also the French and German literature. For instance, in the important book of Birkhoff (Hydrodynamics, Dover Publications, Inc., New York, 1955) "scaling" and "modeling" are used interchangeably and many more examples can be given for the use of the term "model" in the sense of strict similitude. It seems that a deviation from this generally accepted usage would cause more confusion than help. Of course, one could use the term "approximate modeling" in the same way as "distorted model", but the proposed distinction between scaling and modeling appears to be inadvisable.

(2) The definition of scaling given above is rather a definition of theory than scaling and does not follow the lines given in this report and in the literature. Using the above formulation one would rather say: "scaling is seen to exist in physical experiments when phenomena of interest can be

interrelated by linear similitude transformations of their coordinates, regardless of the magnitude of the experiment". This shows that scaling is much more restrictive than theory and one may state that strict scaling almost always fails. Almost all practical scaling attempts are approximations, either excellent, fair or unacceptable ones.

To summarize, it would be desirable to distinguish semantically between the ideal scaling laws and the practical reality by means of two distinctive words which are clearly defined and which will help in communication. It is regrettable that usage in the literature precludes the adoption of the above proposal. As things stand, we hardly have another choice than to use the terms "exact scaling" and "approximate scaling".

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(NOL technical report 61-46)
THE SCALING OF UNDERWATER EXPLOSION PHENOMENA (U), by Hans G. Snay. 30 June 1961. 18p. Task RE01-ZA732/212 9/F008-21-003.
UNCLASSIFIED

The objective of this oral presentation was to summarize the problems and the semantics of scaling in a concise form without use of mathematical developments. The well known cube-root scaling is shown to be a consequence of Mach's similitude, the fourth-root scaling, of Froude's similitude. Requirements which must be satisfied for each of these scaling rules are discussed. Scaling of gravitational effects on underwater explosions (bubble behavior) is not possible by means of model tests conducted in a river or a pond, but requires test tanks in which the air pressure above the water can be reduced or the acceleration of gravity increased.

1. Explosions, Underwater
2. Explosions - Scaling
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II. Snay, Hans G.
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